

MATHEMATICS COURSE DESIGN BASED ON SIX QUESTIONS COGNITIVE THEORY USING HAWGENT DYNAMIC MATHEMATIC SOFTWARE

Liu CunHua¹, Zhou Ying², Ouyang Qunzhuang³, Tommy Tanu Wijaya⁴,

^{1,2,4} School of Mathematics and Statistics, Guangxi Normal University, Guilin, China

³No.18 High School of Guilin

tanuwijayat@gmail.com

Abstract

In the context of "Internet +", using information and intelligence in the education field has become an inevitable trend now. Based on Six-Questions cognitive theory, taking monotonic functions as an example, designing a mathematic learning media using hawgent to solve problems such as "boredom" and "basic understanding" in they present teaching. The goal is to explore the information technology as it can be a reference to optimizing the classroom teaching.

Keywords: *monotonic function; Six-Questions cognitive theory; Hawgent dynamic mathematic software; Students' understanding ability*

In recent years, China's various guide documents have repeatedly pointed to education informatization. On June 2016, the Ministry of Education stated the "13 plan of education informatization" that should be achieved within 5 years. According to the plan, by 2020 the education informatization will be carried out to develop the students in various ways (General Office of Education of the People's Republic of China, 2016). In the 2017 version of "General Standard of High School Mathematics Curriculum" it was suggested that the mathematics teaching activity should use information technology to achieve profound integration between the mathematics curriculum and information technology (Ministry of Education of the People's Republic of China, 2017; Yi et al, 2019). In 2018, the Ministry of Education issued the "Education Action Plan 2.0" that actively promotes the development of "Internet + Education" (General Office of Education of the People's Republic of China 2018).

Looking at the moment, the traditional mathematics concept class is fragmented and superficial teaching (Li Jing, Cai Wen, 2018), which makes it easy for students to only produce "memory" and not "understanding" (Wijaya et al. 2018), which is harmful to the effective learning of students and the cultivation of core literacy . Research shows that teachers proper application of dynamic mathematical techniques can optimize the process and results of students mathematical cognition, reduce their cognitive load, and enhance their effective learning (Xie Dengfeng et al, 2018). However, it may be counterproductive to use dynamic mathematical techniques indiscriminately, without paying attention to teaching principles. Therefore, this article takes "the monotonicity of functions" as an example, and uses the "six-question" cognitive theory as a guide to fuse Hawgent dynamics with classroom teaching, and attempts to explore an effective model of information technology to optimize classroom teaching.

According to the current condition, the teaching method that is used now is very convectional and monotone this makes students to only "memorize" and not "understand"(Li Jing, Cai Wen, 2018; Dini, 2018), which is hinders the development of students' understanding ability and effective learning

(Dewi et al. 2018). Research shows that if teachers use the dynamic mathematical techniques properly, it can optimize the process, student's mathematical cognition result, reduce the cognitive load and improve their effective learning (Xie Dengfeng et al, 2018). However, if we don't pay attention to the teaching principle, using dynamic mathematical techniques can be counterproductive. These articles use the "monotonic function" as an example and use the "Six-Questions cognitive theory" as a guide to integrate hawgent software with classroom teaching. It will also explore various model of information technology that is effective to optimize classroom teaching.

Hawgent dynamic mathematic software

Hawgent is a dynamic mathematics software that has a lot of features. It is flexible and convenient to operate, good teaching function and it has a rich dynamic resources. Hawgent can be use to teach these mathematical knowledge such as geometry, algebra, triangle, probability and etc. The software can show dynamic curve drawing, object trajectory and clearly show the process of mathematical object derivatives. If the software is use properly, it can help conceptual teaching.

Six-Questions cognitive theory

Six-Questions refers to a cognitive theory proposed and is improved by Professor Zhou Ying based on Bernice McCarthy's 4MAT model and Zhu Zhiting's "Five question" taxonomy. The cognitive theory framework are: from where, what is it, what is the connection, how to use, what if it change, think about it (Huang Xiaoyun, Zhou Ying, 2012). Designing a teaching method based on the six-question theory will allow a more complete teaching system and allow students to learn more effectively. A mathematics curriculum model design based on the "Six question cognitive theory" using Hawgent (see figure 1):

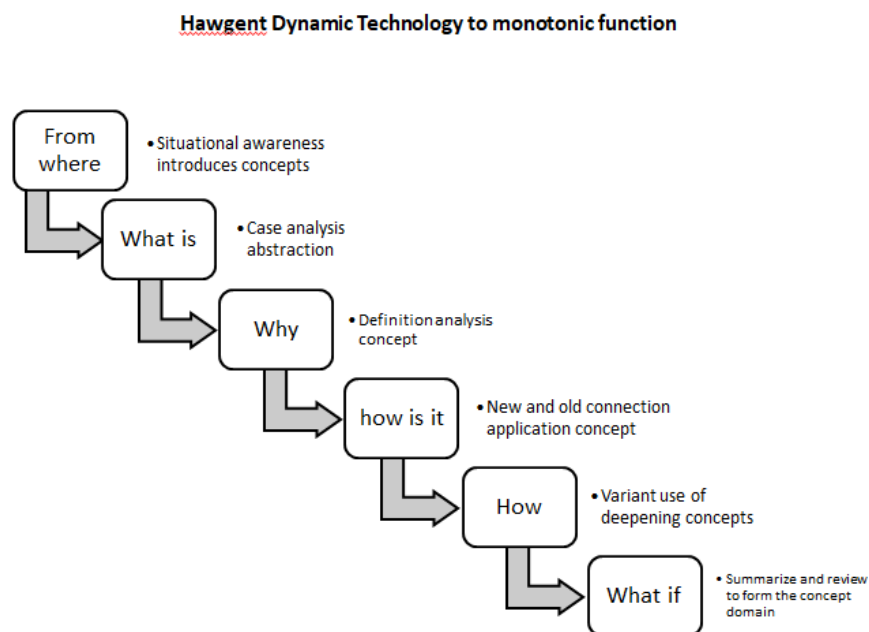


Figure 1. Hawgent Dynamic Technology to Monotic Fuction

Cases Of Mathematical Course Creation Using Hawgent Dynamic Mathematical Software Based On “Six Question” Cognitive Theory

Basic background of Mathematics

The Monotonic Functions is selected from Section 3.2.1, first chapter of a book called "Volume One of General High School Mathematics Textbook". The main content of the chapter is to understand the concept of monotonic function through the combination of "number" and "shape". The image judgment and application definitions of monotonic function proves some monotonic function over a given interval. It emphasizes on the definition of monotony but the application of monotony is difficult. Students' have already understand the basic concepts of function and the concept of monotonic function. If teachers can use Hawgent dynamic mathematic software to solve problems from the Six-Question theory, it will help to highlight the key points and break through difficult points.

the design process of mathematics creation

Based on the analysis of six-question cognitive theory from this chapter, the design process in this chapter is as follows: situation awareness, introduction of concepts (from where); case analysis, abstract concepts (what is it); definition analysis, concept understanding (how to use); connection of concept, concept application (what is the connection); variance application, deepening the concept (what if it change); a review summary, the formation of concept (think about it).

Situational awareness and introduction of concept

The teacher gives a scenario problem which is to observe the images of the following function (Figure 2 and 3) and ask the students to find their changes.

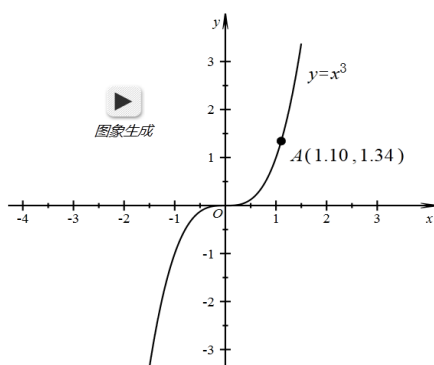


Figure 2. Function graph

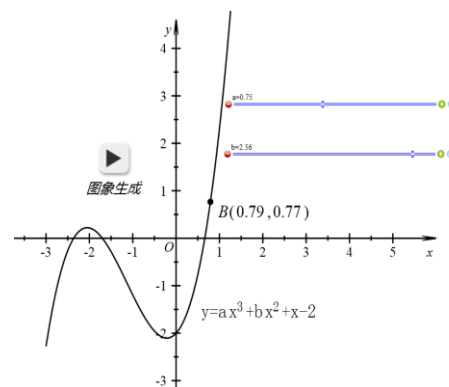


Figure 3. Function graph

The teachers uses Hawgent software to display the graph making process and guide the students to find the change. Figure 3 also has a “variable ruler” which allows students to manipulate the coefficients of the function so that they will be able to get a different function graph. After observing various characteristic of graph change, the teacher summarized the lesson. The teacher explains that when the value of the function increases (or decreases) together with the increase of the independent variable the function is called a monotonic function. This leads to the topic lesson about the concept of monotonic function.

According to the Six-question cognitive theory, this section will discuss the origin of monotonic

function and where it came from. In this stage, students will observe the graph from time to time where the lines can sometimes “rise” and “fall” so that they will be able to know the graph feature of “monotonic function”. Other than that, Hawgent can also dynamically display the graph change and can also be operated by the students which will increase the students’ interest and motivate students to study (Tang Jianlan, Chen Yuan, 2017).

Case analysis, abstract concepts

Firstly, the teacher will introduce the topic and let’s take the function $f(x) = x^2$ as an example for the “rise” and “fall” situation (shown in Figure 4). The teacher guided students to start from the special points and fill in the form. After filling the form, student’s found out that in the $(-\infty, 0)$ interval, the value of $f(x)$ decreases as the value of x increases. As we can see from figure 4, on the $(-\infty, 0)$ intervals we can see that on points x_1, x_2 , the value of $f(x_1) = x_1^2$, $f(x_2) = x_2^2$ where in $f(x_1) > f(x_2)$. With this we can say that in the $(-\infty, 0)$ interval, the value of $f(x)$ has monotonically decreases and similarly in the $(0, +\infty)$ interval, the value of $f(x)$ is monotonically increases.

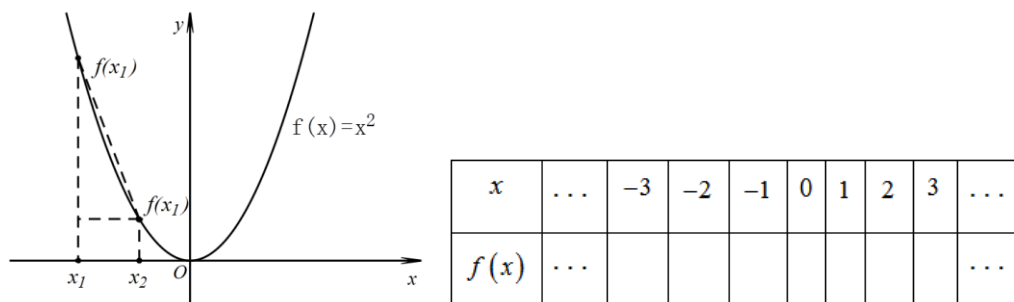


Figure 4. Quadratic function and table Quadratic function

The teacher then let the students to expand their understanding of monotonic function into a general quadratic function. The teacher fist makes a basic increasing function as shown in figure 5. Then the teacher use hawgent and teaches the students about the domain, a certain interval arbitrary points, independent/dependent variables and etc. The same is done with the decreasing function (see Figure 6)

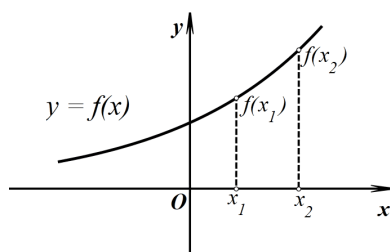


Figure 5. Monotonic increasing function

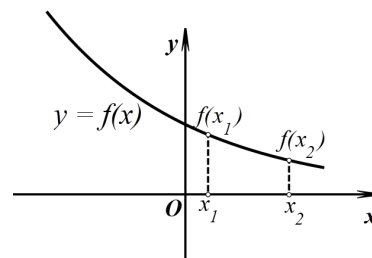


Figure 6. Monotonic decreasing function

According to the six-question cognitive theory, this section will discuss the essential and conceptual knowledge. In this section, students will start with some familiar topics such as quadratic functions. They will also study from special points to general points, from adding to subtracting function and from special function to general functions. In this process, students’ will not only master new knowledge but also learn method of thinking from special to general, analogical reasoning of various numbers and

shapes. It achieved the effect of both “fish” and “fishing” (Tang Jianlan, Chen Yuan, 2017; Chen Nini, Tang Jianlan, 2017).

Analysis of definition and concept understanding

Firstly the teacher ask the students questions about the connotation of the concept such as “Do you have difficulties understanding the definition of monotonically increasing function? What do you think is the key words here?” These questions guides the students to focus on two key points and they are interval D and arbitrary x_1, x_2 .

We then use Hawgent to analyze the two key points by making a quadratic function graph as shown in figure 7. By using hawgent, it allows student to operate the software on their own such as dragging the points x_1, x_2 and the points can be moved deliberately. This means that even if we move the points, it will not effect the independent variables.

Then, ask a question to the students. Set some independent variables on the interval while $\forall x_1, x_2 \in A$, and $x_1 < x_2$. If $f(x_1) < f(x_2)$, can you guess whether the interval D in $f(x)$ is monotonically increasing or decreasing.

Finally, the students were asked to drag the $x_1 x_2$ line segment and they learned that there is a difference in the y-axes value between the right side and the left side values. This shows the locality of the monotonic function.

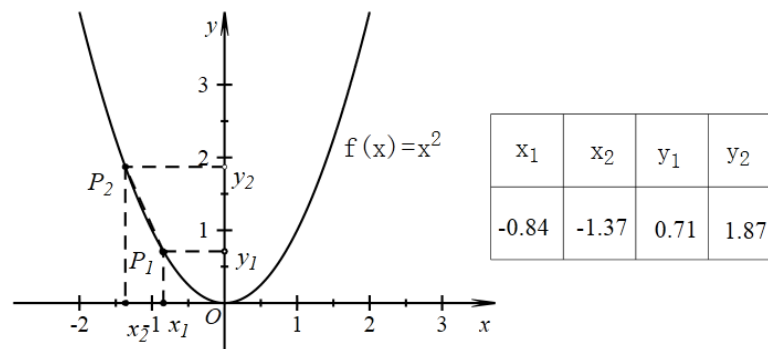


Figure 7. Monotonic Function

According to the six-question cognitive theory, this section will discuss the principal knowledge. Why is the concept so defined? Is it proper if we don't want a certain condition? The depth of students' understanding of the definition and how much they master the core information directly affects the subsequent application of problem solving skills. Therefore, it is necessary to clarify the mathematical thinking process through Hawgent, and turn the thinking process into a visual representation to help understand the concept.

Linking old and new, applying concepts

Firstly the teacher will ask: “Do you remember the functions that you studied during your junior high school years? How do they express function? Who can draw a rough sketch of a function in the blackboard?” The students will draw graphs of functions and list how they express functions as. After that, the teacher will use hawgent to make a standardized graph and explains it.

The teacher will ask “what kind of function is it when $y = kx + b (k \neq 0)$ or when (what is the function” $k < 0$ ”. The teacher will continue the lesson and ask the students to use the definition of monotonic function to prove the conclusion on the question given above.

Lastly, the teachers and the students will review the lesson about the definition of monotonic function and also summarized the basic steps of “assessment on the definition of monotonic function”. The steps are as follow: one, take (assumptions is $x_1 > x_2$); two, subtract ($(f(x_1) - f(x_2))$); three, fix (determine the positive and negative difference); four, assessment (decision result).

According to the six-question cognitive theory, this section will discuss the use of knowledge and how to apply it. This section reviews the basic function knowledge learned during junior high school and connects them to the senior high school function topic. Not only that this will reduce the student’s fear towards theory and application but it will also enhance the students’ cognitive belief. They both uses the Hawgent’s mapping analysis to summarize the basic steps and proves the steps together with the students. At the same time, they reflect and apply the declarative knowledge then transform it into a procedural knowledge to cultivate the students’ mathematical operations and logical reasoning literacy.

Use of variants to deepen concepts

Firstly, the teacher will use hawgent to make a function graph of Boyle’s law. Using the steps of “one take, two minus, three definite and four judgment”, $p = \frac{K}{v}$ (K is constant) and students are required to prove the monotony of the graph. Then the teachers will let the students to do variance training to test the teaching effect.

Variant training 1: Proof that the function $f(x) = -\frac{2}{x}$ in the interval $(-\infty, 0)$ is monotonically increasing

Variant training 2: Proof that the function $y = x + \frac{1}{x}$ in the interval $(1, 8)$ is by definition monotonically increasing

Variant training 3: Define the monotonic function of $f(x) = -\frac{1}{3x} + 6x$ in an interval $(1, 6)$

Lastly, the teacher will use hawgent to make three different function graphs to verify the students’ result. If the students’ did not master the topic very well, the teacher can make a general function graph and let the students do the proofing.

According to the six-question cognitive theory, this section will discuss the methodological knowledge. Through the variance form of teaching, the monotonic function can be proved by the definition method. This can effectively improve the students’ mathematical logical reasoning ability. After explaining about problem-solving, do not be in a hurry to let the student train. Instead, use hawgent dynamic technology combined with the physics model to strengthen the problem-solving steps. Then design three variants with gradients to allow students to have a deeper understanding of the "monotonic concept" and “proofing” so that respond the changes without changing.

Summarize and review, form a concept domain

Firstly, the teacher will show a flowchart of the learning session (figure 8) and ask each learning groups to discuss and summarized the knowledge points as comprehensively as possible in the form of flowcharts, tree diagrams, knowledge network diagrams and concept diagrams.

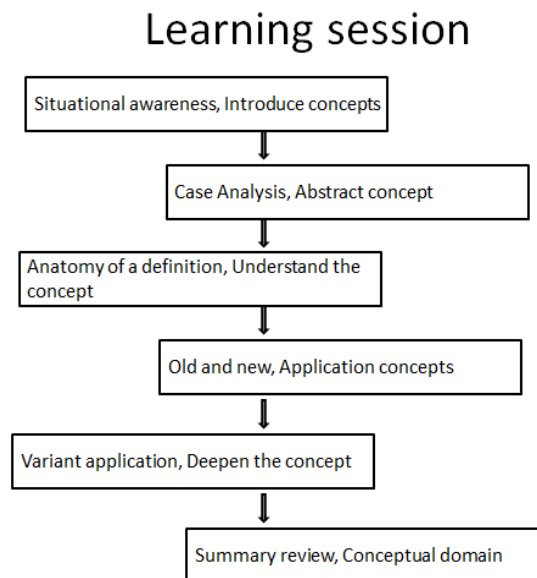


Figure 8. Teaching process flowchart

the teacher will ask each groups' representative to show the result of their group discussion and share the learning experience. The teacher will comment based on their group presentation and uses a mind map (figure 9) to represent the teachers' summary.

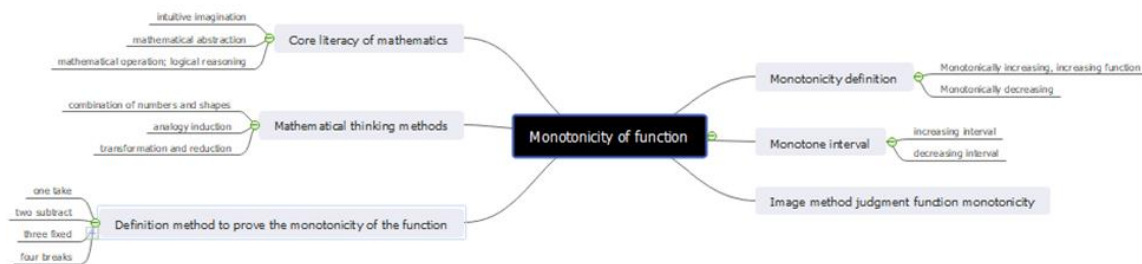


Figure 9. Teachers' summary of the lesson

According to the six-question cognitive theory, this section will discuss the introspective cognitive knowledge. In this part, it will be mainly the review and reflection of the lesson. It will also summarize the lesson in the form of “students’ summary and sharing” and “teachers’ comments and improvement”. This will help the students to refine their knowledge skills, thinking method for this topic, understand the students’ emotional attitude, and strengthen the students’ language skills. The process of refining students’ new knowledge is incorporating the new knowledge in the cognitive system and forming a new concept domain.

CONCLUSION

Six question Cognitive Theory runs through mathematics teaching

The above-mentioned lesson design uses the six question cognitive theory as the main reference for the teaching model, and each section is closely linked with the six question cognitive theory. The

division of labor is clear, and the organizational structure is clear: "from where" traces the source of knowledge, "what is" points out the essence of knowledge, "why" explores the connotation of the concept, "how" exercises the concept application, and "how to" variants deepen the understanding, question "to build a conceptual system. The classroom design under the guidance of "six-he" is progressive and continuously deepening, effectively solving the current situation of fragmented and superficial concept teaching.

Hawgent dynamic technology revitalize mathematics classroom

hawgent dynamic mathematics and other information technologies have emerged in response to the needs of the times, and have become a sharp edge in improving traditional teaching: in the introduction of the classroom, Hawgent has been used to teach students to "sense"; "Fish" and "fishing"; in the analysis of the conceptual link, using hawgent will let the students "know its nature, knowing why it is"; in the application and variant links, with hawgent to explore the process and test results, reduce student recognition knowledge Load. It can be seen that the implementation of teaching in hawgent environment can optimize classroom teaching. The three-in-one teaching of "fish and fish desire" allows students to understand and learn easily.

the core literacy of mathematics can take root

Under the organization of "six question", the help of "hawgent" has enabled students to master the basic knowledge skills and develop the ability methods. At the same time, such a mathematics classroom can also enable the core literacy of mathematics disciplines to be developed: the process of abstraction and definition of the monotonic function of the development of mathematical abstract literacy; the definition of the analogous minus function to obtain the definition of increasing functions, and the development of logic from special functions to general functions Reasoning literacy; develop mathematical literacy with a process that defines the monotonic of a function.

REFERENCES

- Chen Nini, Tang Jianlan. (2017). Mathematics Creative Course Design and Reflection Based on the "Three Worlds of Mathematics": Taking the "monotonicity of functions" as an example. Research on Classroom Teaching of Primary and Middle Schools, 2017 (10): 15- 19.
- Dewi, Senja Noviani, Tommy Tanu Wijaya, Ayu Budianti, and Euis Eti Rohaeti. 2018. "Pengaruh Model Teams Games Tournament Terhadap Kemampuan Pemahaman Matematik Siswa Kelas XI SMK Di Kota Cimahi Pada Materi Fungsi Eksponen." *WACANA AKADEMIKA: Majalah Ilmiah Kependidikan* 2(1): 99.
- Dini, Mentari, Tommy Tanu Wijaya, and Asep Ikin Sugandi. 2018. "Pengaruh Self Confidence Terhadap Kemampuan Pemahaman Matematik Siswa Smp." *Jurnal Silogisme* 3(1): 1–7.
- General Office of Education of the People's Republic of China (2016). "Thirteenth Five-Year Plan" for Education Informatization. From: http://www.moe.gov.cn/srcsite/A16/s3342/201606/t20160622_269367.html.

- General Office of Education of the People's Republic of China (2018). "Education Informatization 2.0 Action Plan". Online: http://www.moe.edu.cn/srcsite/A16/s3342/201804/t20180425_334188.html
- Huang Xiaoyun, Zhou Ying. (2012). The teaching process of "six-he" cognitive chain design: Taking the outer corner of triangle as an example [A]. National Mathematical Education Research Association: China Higher Education Association Higher Teacher Education Research Association Mathematical Education Association, 2012:8.
- Li Jing, Cai Wen. (2018). The Importance of Teaching Mathematics Concepts in Senior Middle Schools [J]. Research on Modern Basic Education. 31 (03): 202-206.
- Ministry of Education of the People's Republic of China. (2017). Mathematics Curriculum Standards for General High Schools. Beijing: People's Education Press, 2017: 2-20.
- Tang Jianlan, Chen Yuan. (2017). Mathematics Creative Class Design Based on the Trinity of "Fish and Fish Desire" to Optimize Teaching Concepts: Taking the Teaching of "Images and Properties of a Linear Function" as an Example. Primary and Middle School Class Teaching Research, 2017 (12): 34-37.
- Wijaya, Tommy Tanu, Neng Suci Septiani Dewi, Indah Retta Fauziah, and M Afrilianto. 2018. "Analisis Kemampuan Pemahaman Matematis Siswa Kelas IX Pada Materi Bangun Ruang." *UNION: Jurnal Ilmiah Pendidikan Matematika* 6(1): 19–28.
- Xie Dengfeng, Chen Nini, Tang Jianlan. (2018). Hawgent Dynamic Mathematics Technology: An "Artifact" for Improving Students' Mathematical Cognition [J]. Classroom Teaching Research in Primary and Middle Schools. (01): 9-15.
- Yi, Lin, Zhou Ying, and Tommy Tanu Wijaya. 2019. "The Trend Of Mathematics Teaching Method Has." 03(02): 471–80.